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FORM FACTORS OF THE DECAY $\pi^+ \to e^+ e^- \nu e^+$

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The decay $\pi^+ \to e^+ e^- \nu e^+$ is studied in the quark model of superconductivity type. The calculated form factors R and the relation $\xi = R/F_V$ are in a good agreement with experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Формфакторы распада $\pi^+ \to e^+ e^- \nu e^+$

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В кварковой модели сверхпроводящего типа описан распад $\pi^+ \to e^+ e^- \nu e^+$. Полученные оценки для формфактора R и отношение $\xi = R/F_V$ согласуются с экспериментом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

In[1], based on the Quark-Model of Superconductor Type[2] the form factors (vectorial and axial) were calculated for the decay $\pi^- \to e^- \bar{\nu} \gamma$. It has been shown that for a_1-mesons, the relation $\gamma = F_A/F_V = Z^{-1}$ becomes less than one ($Z^{-1} = 0.7$), where $Z^{-1} \simeq 1 - 6m_u^2/m_{a_1}^2$, $m_u = 280$ MeV is the mass of the u-quark in QMST and $m_{a_1} = 1275$ MeV is the mass of the a_1-meson. This result for γ is in good agreement with experimental data[3-5]. Some time ago new experimental data were published by the group SINDRUM[5] for the decay $\pi^+ \to e^+ e^- \nu e^+$, where besides the well-known form factors F_A and F_V also the axial form factor R appeared, connected with the momentum of the e^+e^--pair. Therefor it is useful to calculate R and the relation $\xi = R/F_V$ that is also determind experimentally. In[5] the following results are given for the form factors:

$$F_V = 0.029^{+0.019}_{-0.014}, \quad F_A = 0.018^{+0.015}_{-0.012}, \quad R = 0.063^{+0.028}_{-0.016}, \qquad (1)$$

$$\gamma = 0.7 \pm 0.5, \quad \xi = 2.3 \pm 0.6.$$

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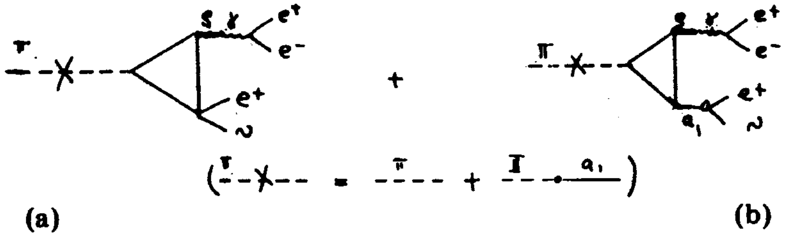
Using [6] for determining the structure part of the amplitude $\pi^+ \to e^+ e^- \nu e^+$ one gets:

$$T^{s}_{(\pi^+ \to e^+ e^- \nu e^+)} = i e^2 G \frac{1}{k^2 m_\pi^2} F_\pi \ell_\alpha^{em} \{ F_\nu \epsilon_{\alpha\beta\mu\nu} k^\mu Q^\nu + \\ + i F_A (Q_\alpha k_\beta - g_{\alpha\beta} Qk) + iR(k_\alpha k_\beta - g_{\alpha\beta} k^2) \} \ell_\beta^{w}, \tag{2}$$

where G is the Fermi constant, e is the charge of the electron; F_π, the constant of pion-decay $F_\pi = 93$ MeV; ℓ_α^{em}, the electromagnetic current $\ell_\alpha^{em} = \bar{u}_e(p_3)\gamma_\alpha u_e(-p_2)$; ℓ_β^w, the weak current $\ell_\beta^w = \bar{u}_\nu(p) \times \times \gamma_\beta (1-\gamma_5) u_e(-p_1)$; Q, the momentum of the pair $e^+\nu$ and k is the momentum of the pair e^+e^-. Only the diagrams in the Figure contribute to the amplitude (2), where the a_1-meson playes an important role as an intermediate particle. If only diagram (a) is used, then F_A and F_V will be equal:



Diagrams of the decay $\pi^+ \to e^+ e^- \nu e^+$.

$$F_A^{a)} = F_V^{a)} = \frac{1}{8\pi^2} \frac{m_\pi^2}{F_\pi^2}, \qquad \gamma = 1. \tag{3}$$

Using also diagram (b) we finally get for the form factors F_V and F_A the expression[1]: $F_V^{a+b)} = F_V^{a)}$, $F_A^{a+b)} = F_A^{a)} Z^{-1}$, $\gamma = Z^{-1}$. This result agrees with the experimental (see (1)). It will be shown that the influence of diagram (b) is the same for the form factor R : $R^{a+b)} = R^{a)} Z^{-1}$. The form factor R is absent in the decay $\pi^- \to e^- \bar{\nu} \gamma$ because $k^2 = 0$ for the photon, but when the e^+e^--pair appears, $k^2 \neq 0$ and there occurs a k^2-dependence in the amplitude (2) (form factor R).

For the calculation of R, the amplitude for the decay $a_1 \to \pi\rho$ will be used, taken from[1]:

$$T^{\mu\nu}_{(a_1 \to \pi\rho)} = i\frac{g_\rho^2}{8\pi^2 F_\pi}\{p^\mu k^\nu - g^{\mu\nu}\, pk + [1 + 2Z(\frac{2\pi F_\pi}{m_{a_1}})^2]\, g^{\mu\nu} k^2\}. \quad (4)$$

Here g_ρ is the constant of the decay $\rho \to 2\pi$ with $g_\rho^2/4\pi \approx 3$, p and k are the momenta of the pion and the ρ-meson. Before using this expression for calculating the axial part of the amplitude (2) it is necessary to substitute $p \to -p$. If we use only diagram (a) for the calculation of R, we get:

$$R^{a)} = \frac{1}{8\pi^2}\frac{m_\pi^2}{F_\pi^2} 2(1 + Z(\frac{2\pi F_\pi}{m_{a_1}})^2) = F_V\, 2(1 + Z(\frac{2\pi F_\pi}{m_{a_1}})^2)$$

but, finally using both the diagrams, (a) and (b), we get for the form factor R and the relation ξ the following expressions:

$$R^{a+b)} \equiv R = F_V 2(Z^{-1} + (\frac{2\pi F_\pi}{m_{a_1}})^2) = 0.053, \quad \xi = 2(Z^{-1} + (\frac{2\pi F_\pi}{m_{a_1}})^2) = 1.83.$$

These theoretical results agree with the experimental data (1) (remind that theoretical values for F_V and F_A are: $F_V = 0.029$ and $F_A = 0.020$ /1/). At the end of this short report we wish to compare our results with those of the Quark-Virton Model /7/:

$$F_V = 0.041, \quad F_A = 0.023, \quad R = 0.087, \quad \gamma = 0.56, \quad \xi = 2.1.$$

In that model there was also taken into account the a_1-meson.

A comparison of the values we have obtained for the form factors F_V, F_A and R shows that our prediction in the QMST is slightly better than that in /7/ (see (1)). The expressions for the form factors in the QMST are analytically very simple so it is easy to compare them with the results of other models, for example, with the standard Quark Models $F_V = F_A = m_\pi^2/8\pi^2 F_\pi^2$ /8/. It is possible to show here that the intermediate a_1-meson plays a very important role only for an exact calculation of the form factor F_A and the ratio γ which is in a good agreement with the experimental data (γ becomes less than one ($\gamma = 0.7$) under the influence of the a_1-meson). The value of R slightly changes after taking into account of the a_1-meson.

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